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Writing Assignments in an Abstract Algebra Course

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Including writing assignments in my algebra courses came about due to my college's (at the time Saint Mary's College) and my own belief that students should "write across the curriculum" and in particular in their majors' courses. Many papers have discussed the importance and benefits of writing in math courses. In this paper I will discuss three specific writing assignments that I have designed and used and some observations about each.

The first time I taught Abstract Algebra I used the sample writing assignment described by Anne Brown in Chapter 29 of *Using Writing to Teach Mathematics*. My subsequent assignments were modeled after this one. However, I have found it is not necessary to give the students a detailed outline of the paper.

When designing my own writing projects, I have three main goals: the assignment can be broken down into parts; the assignment relates to the mathematics covered in the course but also introduces new topics; and the assignment is not too mathematically challenging so that the students can concentrate on the writing and not get bogged down in the mathematics.

They have a chance to rewrite and correct the mathematics before the final paper is turned in. This has allowed the students to concentrate on their writing when they work on the final paper.

The exercises in the assignments are standard. Although students are already expected to write proofs in complete sentences, these assignments go beyond normal homework exercises. The students are required to include all relevant definitions, some examples, lemmas, and finally the proof of a theorem. The papers are usually five to ten pages long.

Breaking the Assignment Down into Parts

There are several benefits to breaking the assignment down into parts. I noticed in my earlier writing assignments in which I did not do this that the students were afraid to sit down and start writing. They did not think that the words "write" and "math" belonged in the same sentence. They viewed the assignment as some enormous project hanging over their heads and tended to procrastinate. Thus, one or two nights before the project was due, they were panicking and frantically trying to write a math paper. By breaking the assignment down into smaller pieces, they could work on one portion at a time. The students have commented on course evaluations that they appreciate the project being structured in this manner.

Another positive aspect of breaking the assignment down into parts is that the students get feed back along the way. They have a chance to rewrite and correct the mathematics before the final paper is turned in. This has allowed the students to concentrate on their writing when they work on the final paper. I no longer require a draft of the final paper since these earlier exercises are drafts. However, the students are free to turn in drafts that I will make comments on, but not grade.

The first part of the assignment is to write a short paper introducing the necessary terms, giving examples, and explaining notation. The middle parts of the paper tend to introduce new topics tangential to the course and involve some proofs that will be used in the last part of the assignment. This last assignment is the goal of the entire paper and usually involves proving a theorem.

The parts of the assignment are designed so that writing the final paper only requires putting the components together with transitions and an introduction and conclusion. I have found that I have to repeatedly remind students of the importance of transitional sentences and paragraphs. They think that I am being funny when I say that the final paper should flow like a novel.

DISTRIBUTING THE ASSIGNMENT TO THE STUDENTS

Copies of three of my assignments as they were given to the students can be found at the end of this paper. When I make the assignment, I give them all of the parts as well as a written explanation of how the paper will be graded. At this time I also give them all due dates. These due dates tend to be one week apart with a couple of weeks between the last part and the final paper.

GRADING

As mentioned above I give the students some written guidelines on how the paper will be graded when it is assigned. (A copy of this is also attached.) These are based on the departmental guidelines for evaluating writing used by the Mathematics Department at Saint Mary's College. I grade each part of the assignment separately and include these scores in the student's homework grade. The student also receives a grade for the final paper.

COMMENTS ABOUT THE ASSIGNMENTS

The first two assignments were used in a sophomore level Algebraic Structures course. The third assignment was used in a junior/senior level second semester Abstract Algebra course.

One problem I encountered with the first assignment listed was that I gave the names of the new concepts defined in part (2). For some reason the students spent time worrying about the terms "center" and "centralizer" and lost sight of the fact that they were just trying to prove a set was actually a subgroup. Originally, I included the new terms so that they would be familiar with them and also so that they could look them up in reference books. I do not think that they used resources other than their own text even though they were told that they could. In fact, the proof that the center of a group is a subgroup of the group is in the text book we used in a chapter that we did not cover. After they turned in their papers, I pointed this out to them. No one had even noticed this.

Notice in the second assignment I decided not to give the names of the new concepts. This seemed to work out better from the students' point of view. When these ideas came up naturally in later class lectures, I introduced the formal terminology. Parts (1b) and (1c) were included to reinforce the concepts of cyclic and abelian groups. However, I might exclude these from the assignment in the future because neither is required to prove the theorem listed in part (4) and the students seemed distracted by these parts.

The third assignment was probably the most successful. Even the students who had done writing assignments for me in other courses mentioned that this was their favorite. I liked this particular assignment because it tied together so many topics: ring, commutative ring, unity, ideal, and field. Obviously, the mathematics is not difficult, but it requires a clear understanding of the topics and some thinking beyond the course work. Writing the paper helped the students review for the test over this material. One of the test questions was the converse of the theorem proven in the paper. When we went over the test in class, I pointed out that with the paper and that test question we now had an if and only if statement. Linking the test and the paper made the paper a more natural part of the course from the students' perspective.

Although it can be difficult at times to design writing assignments that meet all of my goals, it can be done with some careful planning before the semester begins. I have found these projects to be very worthwhile and will continue to incorporate them into my courses.

CONCLUSION

Although it can be difficult at times to design writing assignments that meet all of my goals, it

can be done with some careful planning before the semester begins. I have found these projects to be very worthwhile and will continue to incorporate them into my courses.

REFERENCES

1. Bloch, Norman J., *Abstract Algebra with Applications,* Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1987.

2. Brown, Anne E., "Writing to Learn and Communicate Mathematics: An Assignment in Abstract Algebra", in *Using Writing to teach Mathematics,* Andrew Sterrett (ed.), Mathematical Association of America, Washington, DC, 1990.

3. Gallian, Joseph A., *Contemporary Abstract Algebra,* D.C. Heath and Company, Lexington, Massachusetts, 1990.

APPENDIX A

MATHS 311
AUTUMN 1991
WRITING ASSIGNMENT

You may not work together on this assignment. However, you may see me for help.

1. Write and type a 1-2 page paper explaining the concepts of group, abelian group, subgroup, etc. Include some examples. Explain notation.

DUE: Thursday, Sept. 26

2.a) Let G be a group. Define the center of G to be
$Z(G) = \{ a \in G \mid ax = xa \text{ for all } x \in G\}$.
Prove that Z(G) is an abelian subgroup of G.

b) Let G be a group and $a \in G$, a fixed element. Define the centralizer of a in G to be
$C(a) = \{ g \in G \mid ga = ag\}$.
Prove that C(a) is a subgroup of G.

DUE: Thursday, Oct. 10

3. Prove the following theorem.

THEOREM: $Z(G) = \cup_{a \in G} C(a)$. (This means the intersection of all subgroups of the form C(a).)

DUE: Thursday, Nov. 7

APPENDIX B

MATHS 311
SPRING 1992
WRTTING ASSIGNMENT

You may not work together on this assignment. However, you may see me for help.

1. Write and type a 1-2 page paper explaining the concepts of group, abelian group, cyclic group, subgroup, etc. Include some examples. Explain notation.

DUE: FRIDAY, FEBRUARY 7

2. Let G be a group and H a subgroup of G. For any x in G, define $x^{-1}Hx = \{ x^{-1}hx \mid h \in H \}$.
a) Prove: $x^{-1}Hx$ is a subgroup of G
b) Prove: If H is cyclic, then $x^{-1}Hx$ is cyclic.
c) Prove: If H is abelian, then $x^{-1}Hx$ is abelian.

DUE: FRIDAY, FEBRUARY 14

3. Let G be a group and let H be a subgroup of G. Define
$N(H) = \{x \in G \mid x^{-1}Hx = H\}$.
Prove that N(H) is a subgroup of G.

DUE: FRIDAY, FEBRUARY 21

4. Let H be a subgroup of G and x an element of G.

Prove $N(x^{-1}Hx) = x^{-1}N(H)x$.

DUE: FRIDAY, FEBRUARY 28

The final paper is to put all of the above parts together into one logically connected body.

DUE: FRIDAY, MARCH 20

APPENDIX C

MATH 354
SPRING 1990
WRITING ASSIGNMENT

1. Write a 1-2 page paper explaining the concepts of ring, commutative ring, unity, units, ideal, field, etc. Include some examples. Explain notation.

2. For any element a in a ring R, define <a> to be the smallest ideal of R that contains a. If R is a commutative ring with unity, show that <a> = aR = {ar | r ∈ R).

3. Let R be a commutative ring with more than one element. Prove that if for every non-zero element a of R, we have aR = R, then R is a field.

PAPER: Prove the following theorem.

Theorem. Let R be a commutative ring with unity. Suppose the only ideals of R are {0} and R. Show that R is a field.

DUE DATES:	EXERCISE 1	Feb. 4
	EXERCISE 2	Feb. 11
	EXERCISE 3	Feb. 18
	DRAFT (TYPED)	Mar. 8
	FINAL PAPER	Apr. 8

APPENDIX D

Your grade on the paper will be based on the following:

Knowledge of math
- insight
- clear, concise, complete
- examples
- notation, symbols, variables
- accurate

Originality
- independent thought
- audience

Organization
- orderly, logical manner
- smooth transitions
- intro and conclusion

Vocabulary
- math vocabulary
- word choice

Sources
- references in text
- bibliography

Grammar

And This is the Tale That Xeno Told

Sandra Z. Keith
St. Cloud State University

Remember great Achilles, most handsome, lithe and tall...
You sing of your Olympians—why he'd outrun them all!
Achilles favored turtle soup, but to obtain that feast,
He had, you understand, to stoop and catch the beast.
Imagine! Great Achilles—was challenged by a turtle!
Said Turtle: "you can't catch me!", but Achilles merely chortled;
"I walk ten times as fast as you—I'll catch you near or far!"

He said he'd give the beast a start; he'd yield a thousand yards.
He was not man but demi-god, and had to save his face,
For all the gods had gathered there to watch that famous race.